© copyright Ayman Badawi 2015

# MTH 221 Linear Algebra Spring 2015, 1–7

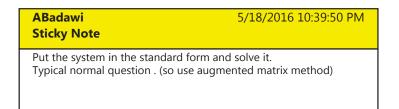
# Final Exam, MTH 221, Spring 2015

Ayman Badawi

**QUESTION 1.** (10 points). Find the solution set for the the following system of linear equations

$$x_1 + 2x_2 = 1 - x_3$$
$$x_2 = 2 - x_3$$
$$x_1 + 3x_2 = 3 - 2x_3$$





## **QUESTION 2.**

(4 points). Given  $S = span\{(1,0,0,1), (1,1,0,1), (1,1,1,1)\}$ . Use Gram-Schmidt Algorithm to find an orthogonal basis for S.



 ABadawi
 5/18/2016 10:09:06 PM

 Sticky Note
 5/18/2016 10:09:06 PM

 Typical normal question (just use my class notes). No ideas/ only straight forward calculation

## QUESTION 3. (10 points) Let

$$A = \left[ \begin{array}{rrrr} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{array} \right]$$

(i) Find the inverse matrix of A if it exists.

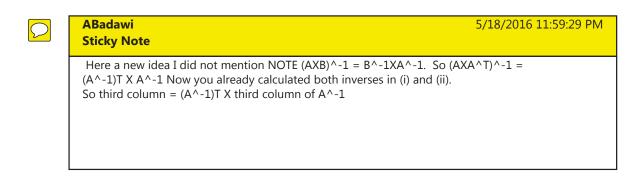
ABadawi Sticky Note 5/18/2016 11:36:51 PM

Again:)) Nothing new! straight forward calculations. Just do [A | I\_3] cook it until you get [I\_3 | A^{-1}]

(ii) Find the inverse matrix of  $A^T$  if it exists.

ABadawi Sticky Note	5/18/2016 11:58:29 PM
Note No calculation needed here. You already calculated A^{-1} in (i) So (A^T)^{-1} = (A^{-1})T	

(iii) Find the third column of the inverse matrix of  $AA^T$  if it exists.



**QUESTION 4. (6 points).** Let  $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$  such that  $T(a, b, c) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ . Then T is a linear transformation of the point of the

mation (DO NOT SHOW THAT).

(i) Find dim(Range) and write the Range as a span of a basis.

 $\mathcal{D}$ 

ABadawi Sticky Note	5/18/2016 11:30:37 PM
The given matrix , say M, is the standard matrix representation of T. So Range = Col(M)/ see class notes	

(ii) Does the point Badawi = (4, 5, 0) belong to the Range of T? If yes, find a point, say Ayman = (a, b, c), such that T(Ayman) = Badawi

ABadawi Sticky Note	5/18/2016 11:31:30 PM
To find the point Ayman = (a, b, c) Solve the system of linear equations $MX = (4, 5, 0)^T$ (M is the given standard matrix rept). If there is a solution, then (4, 5, 0) does belong to the range. If no solution, then (4, 5, 0) does not belong to the range	)

**QUESTION 5.** (4 points). Imagine that K is a subspace and  $\{k_1, k_2\}$  is a basis for K. Is  $\{k_1, k_1 + k_2\}$  a basis for K? Convince me (briefly) that your answer is acceptable.

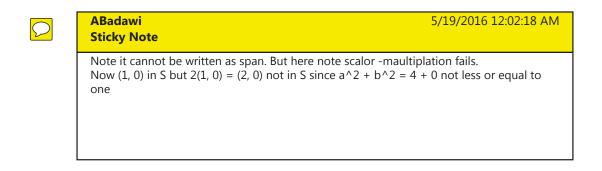
5	ABadawi         5/19/2016 11:11:38 AM           Sticky Note         5/19/2016 11:11:38 AM
	We only need show K_1, K_1 + K_2 independent Set $a_1K_1 + a_2(K_1 + K_2) = 0$ . Show $a_1 = a_2 = 0$ (normal zero). So $(a_1 + a_2)K_1 + a_2K_2 = 0$
	(additive identity O). Since K_1, K_2 independent, $a_1 + a_2 = 0$ and $a_2 = 0$ . Hence $a_1 = 0$ as well. Done

**QUESTION 6.** (9 points) For each of the below, if the subset S is a subspace, then rewrite it as a span of some basis, and tell me its dimension. If not a subspace, then give a counter-example.

(i)  $S = \{(a, b) \in \mathbb{R}^2 \mid (a, b) \text{ is orthogonal to } (2, -1)\}$ 

ABadawi Sticky Note	5/18/2016 10:21:06 PM
Note S = {(a, b)   2a - b = 0, a, b in R}. Hence S = {(a, 2a)   a in R} = span {(1, 2)}.	

(ii)  $S = \{(a, b) \in \mathbb{R}^2 \mid a^2 + b^2 \le 1\}$ 



(iii)  $S = \{f(x) \in P_3 \mid f(0) = 3\}$ 

ABadawi Sticky Note	5/18/2016 11:19:59 PM
Note that $S = \{a_2x^2 + a_1x + 3\}$ not equal spar + 3 in S but if we add them , then we get $x^2 + x + 6$ and if we substitute 0 fo	

Ayman Badawi

## **QUESTION 7.** (11 points).

A =	a	b	c	d	e
	f	g	h	i	j
A =	k	l	m	n	0
	p	q	r	s	t
	u	v	w	x	<i>y</i>

and suppose that  $det(A) = \pi$ .

(i) Find det
$$(A^{-1})$$
,  $det(2A^T)$ ,

ABadawi Sticky Note	5/18/2016 11:04:47 PM
TRIVIAL QUESTION	JUST MY GIFT to all /I mean ALL

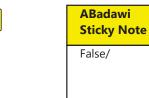
(ii) Find the determinant of 
$$B = \begin{bmatrix} 1 & b & c & d & e \\ 0 & 1 & h & i & j \\ 0 & 0 & 1 & n & o \\ 0 & 0 & 0 & 1 & t \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$
,

For the matrix B above, what are the eigenvalues of B? Assume that B is is diagnolizable (Do not show that), for each eigenvalue a of B find  $dim(E_a)$ 

	1	b	c	d	e	]
(iii) Find the determinant of	0	1	h	i	j	
(iii) Find the determinant of	0	0	1	n	0	.
	0	0	0	1	t	
	1	b	c	d	3 + e	

QUESTION 8. (16 points) Determine whether each statement is true or false and give a brief justification for your answer (should not exceed two <u>CLEAR MEANINGFUL</u> lines)

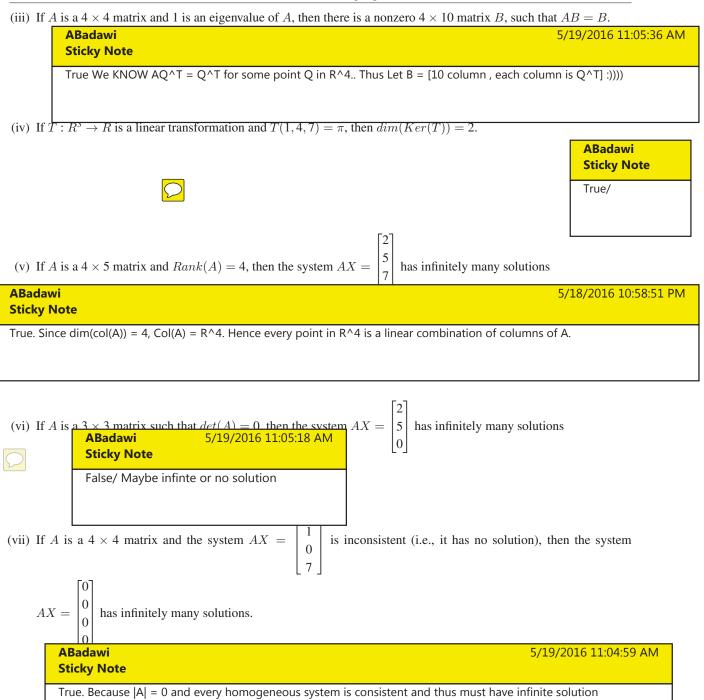
(i) If A is a  $3 \times 3$  invertible matrix, then A is diagnolizable.



(ii) It is impossible to construct a linear transformation  $T : \mathbb{R}^2 \to \mathbb{R}^4$  such that dim(Range(T)) = 3. **ABadawi**  $5/19/2016 \ 11:20:56 \ AM$ 

ABadawi Sticky Note

True, since dim(Ker) + dim(range) = dim (domain)



(viii) If A is a 4 × 4 matrix and $C_A(x) = x^2(x-3)^2$ , then the system $AX =$	0 0 0 0	has infinitely many solutions.
--	------------------	--------------------------------

many solutions
ma

#### **Faculty information**

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates. E-mail: abadawi@aus.edu, www.ayman-badawi.com

7

$$\boxed{Q1} \text{ Let } M = \begin{bmatrix} -1 & -2 & 1 & 3 \\ 1 & 0 & 1 & -1 \\ 2 & 1 & 2 & 1 \end{bmatrix}. \text{ Then the COMPLETE reduced form of } M \text{ is}$$

$$A) \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & -2 & 2 & 2 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$B) \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$C) \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$D) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$E) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

5

Q2 Let M be a  $n \times n$ - matrix such that  $det(M) \neq 0$ . Which of the following statements is **always true** 

- A) M is diagonalizable
- B) M has n distinct eigenvalues
- C) 0 is an eigenvalue of M
- D) It is possible that 0 is an eigenvalue of  $A^T$
- **E)** All eigenvalues of M are nonzero

$$\boxed{Q3} \text{ If } A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ is such that det } A = 4 \text{, then the determinant}$$
$$\begin{vmatrix} a - 2d & b - 2e & c - 2f \\ \frac{1}{2}g & \frac{1}{2}h & \frac{1}{2}i \\ 2d & 2e & 2f \end{vmatrix} \text{ is equal to}$$
A) 8  
B) 4  
C) 2  
D) -8  
E) -4

7

Q4 Consider the following subsets of  $\mathcal{P}_3$ :

$$R = \{ f(x) \in \mathcal{P}_3 : f'(2) = 0 \}, \ S = \{ f(x) \in \mathcal{P}_3 : f(1) \ge 0 \}$$
  
and  $T = \{ f(x) \in \mathcal{P}_3 : f(x) + f'(x) = 0 \}.$ 

Which of these subsets is a subspace of  $\mathcal{P}_3$ ?

A) R, S, and T

**B)** R and T only

C) T only

D) S only

E) R only

Q5 Recall that a square matrix A is said to be symmetric if  $A^t = A$ . If A is a square matrix, then

- A)  $AA^t$  and  $A A^t$  are symmetric
- B)  $A + A^t$  and  $A A^t$  are symmetric
- $\bigcirc$   $AA^t$  and  $A + A^t$  are symmetric
- D)  $AA^t$ ,  $A + A^t$  and  $A A^t$  are symmetric
- E)  $AA^t$ ,  $A + A^t$  and  $A A^t$  are not symmetric

Q6 Which of the following sets is a basis for  $\mathcal{P}_3$ 

A)  $\{1 + x + x^2, 1 + 2x + 2x^2, -2 - 3x - 3x^2\}$ B)  $\{1 + x + x^2, x + x^2, 2\}$ C)  $\{x + x^2, x + 1, -x^2 + 1\}$ D)  $\{1 + x + x^2, x + x^2, x^2\}$ E)  $\{1, 1 + x + x^2\}$  

 Q7 If the point  $(1, a, b) \in span\{(1, 1, 0), (2, 1, 1), (2, 3, -1)\}$ . Then

 A) a = 0 and b = 2 

 B) a = 1 and b = 1 

 C) a = 1 and b = -1 

 D) a = 2 and b = 1 

**E)** a = 2 and b = -1

 $\boxed{Q8} \text{ Let } T : \mathbb{R}^4 \to \mathbb{R}^3,$ 

$$T(a, b, c, d) = \begin{bmatrix} -3 & 1 & 3 & -2 \\ 1 & 1 & -3 & 4 \\ 1 & 3 & -5 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

A) dim Range(T) = 2

- B) dim Ker(T) = 0
- C) (-3,3,5) is not in Range(T)
- $\mathbf{D} Range(T) = \mathbb{R}^3$
- E)  $Ker(T) = span\{(0, 1, 1, 0)\}$

Q9 Let  $T : \mathbb{R}^n \to \mathbb{R}^m$  be a linear transformation such that  $Kert(T) = \{(0, 0, ..., 0)\}$ and  $Range(T) = R^m$ . Let M be the standard matrix representation of T. then

A) n < m and dim(Row(M)) = n

B) n > m and dim(Col(M)) = m

C) It is possible that det(M) = 0.

 $\mathbf{D} n = m$ 

E) It is impossible that  $M = M^T$ 

Q10 Let  $T : R^2 \to P_2$  be a linear transformation such that T(1, 1) = x and T(-1, 1) = 2. Then T(0, 4) =

- **A)** 2*x* + 4
- B) *x* + 4
- C) 2*x* + 2
- D) 4
- E) 4*x* + 8

Q11 The following system of linear equations:

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 0 \end{bmatrix}$$

A) has a unique solution

B) has infinitely many solutions

C) has 
$$\begin{bmatrix} 1\\1\\1 \end{bmatrix}$$
 as a solution

**D**) has no solution

E) has 
$$\begin{bmatrix} -5\\0\\0 \end{bmatrix}$$
 as a solution

Q12 Let  $T : \mathbb{R}^2 \to \mathbb{P}_2$  be a linear transformation such that T(a, b) = (a + 3b)x + (2a + 6b). Then the fake standard matrix representation of T is

$$\begin{array}{c} A) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \hline B & \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \\ C) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ D) \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix} \\ E) \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

Q13 Let T as above then: A)  $Ker(T) = \{(0,0)\}, Range(T) = span\{x\}$ 

B)  $Ker(T) = \{(0,0)\}, Range(T) = Span\{x+2\}$ 

C)  $Ker(T) = span\{(1, -3)\}, Range(T) = Span\{x + 2\}$ 

D)  $Ker(T) = span\{(-3, 1)\}, Range(T) = Span\{x\}$ 

E)  $Ker(T) = span\{(-6,2)\}, Range(T) = Span\{x+2\}$ 

Q14 Let  $M = \begin{bmatrix} a^2 & a^3 \\ 1 & a^4 \end{bmatrix}$ . Which of the following statements is **always true** 

A) When *M* is invertible, 
$$M^{-1} = \begin{bmatrix} \frac{a}{a^3 - 1} & \frac{-1}{a^3 - 1} \\ \frac{-1}{a^3(a^3 - 1)} & \frac{1}{a(a^3 - 1)} \end{bmatrix}$$

B) det M = 0 only if a = 1

C) M is invertible only if  $a \neq 0$ 

D) When *M* is invertible, 
$$M^{-1} = \begin{bmatrix} \frac{1}{a(a^3 - 1)} & \frac{-1}{a^3 - 1} \\ \frac{-1}{a^3(a^3 - 1)} & \frac{a}{a^3 - 1} \end{bmatrix}$$

E) M is row equivalent to  $I_2$ 

### **Faculty information**

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates. E-mail: abadawi@aus.edu, www.ayman-badawi.com